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Analysis Of A Dusty Wall Jet

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Abstract

An analysis is given for the entrainment of dust into a turbulent radial wall jet. Equations are solved based on incompressible flow of a radial wall jet into which dust is entrained from the wall and transported by turbulent diffusion and convection throughout the flow. It is shown that the resulting concentration of dust particles in the flow depends on the difference between the applied shear stress at the surface and the maximum level of shear stress that the surface can withstand ($\propto \rho_d a_g D$) i.e. the pressure due to the weight of a single layer of dust. The analysis is expected to have application to the downflow that results from helicopter and VTOL aircraft.

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List of Symbols

a_g	acceleration due to gravity
b	jet half width
c	concentration of dust particles
c_c	compaction factor
C_f	shear stress coefficient
C_f^*	allowable shear stress coefficient
D	dimension of dust particles
F	dimensionless stream function
G	dimensionless concentration of dust particles
J	radial wall jet momentum
k	constant (= .8814)
K	constant (= .078)
K_1	constant (= $\frac{K^2}{2k^2} = .004$)
\dot{m}	rate of mass transport
p	pressure
r, ϕ, z	cylindrical coordinates

u, v, w	time mean velocity components in the r, ϕ and z directions
u', v', w'	velocity fluctuations in the r, ϕ and z directions
\overline{u}	average velocity
α	angle of repose of dust particles
ε	eddy viscosity coefficient
η	dimensionless distance ($= \frac{1}{\sqrt{2K_1}} \frac{z}{r}$)
μ	($= \frac{1}{K} (C_f - C_f^*)$)
ν	kinematic viscosity
ρ	density
τ	shear stress in absence of entrainment
τ^*	allowable surface stress ($= \frac{1}{2} \rho_d a_g D$)

Subscripts

a	air
d	dust
m	maximum velocity point
w	wall
∞	infinity
o	no entrainment

Chapter 1

Introduction

1.1 Motivation

Helicopters and VTOL aircraft necessarily operate near the ground during landing and takeoff and in nap-of-the-earth flight; they can interact with the ground causing both ground erosion and dust entrainment into the vehicle flowfield. Such effects as helicopter blade erosion, engine ingestion of dust particles and loss of visibility may result, while damage due to dust impingement may occur to objects on the ground in the vicinity of the vehicle.

1.2 Previous Work

The flowfield produced by a circular turbulent jet impinging normally on a flat plate has been studied by Bradshaw and Love [1], Bradbury [2], Poreh, Tsuei and Cermak [3], and Beltaos and Rajaratnan [4]. These studies have not taken the entrainment of dust particles into consideration. Most of these authors studied the mean pressure and velocity fields, as well as their turbulent characteristics, either in the impact region or in the radial wall jet [5].

Borges and Viegas [5] used an "erosion technique" to study the effects of an impinging jet on a flat plate covered with a thin layer of uniform particles.

1.3 Current Approach

The analysis given here treats the entrainment of dust into a radial wall jet and serves as a preliminary model for the more complex case of the flowfield around a helicopter or a VTOL aircraft. The distribution of dust particles in the radial wall jet is determined in terms of the radial wall jet momentum, the surface friction coefficient and the resistance of the surface to shear stress which depends on the density of the dusty material and the diameter of the particles.

Chapter 2

Analysis

2.1 Model

A radial wall jet is essentially a free radial jet emanating from a point source of momentum and flowing over a surface thereby forming a boundary layer as depicted in Figure 2.1.

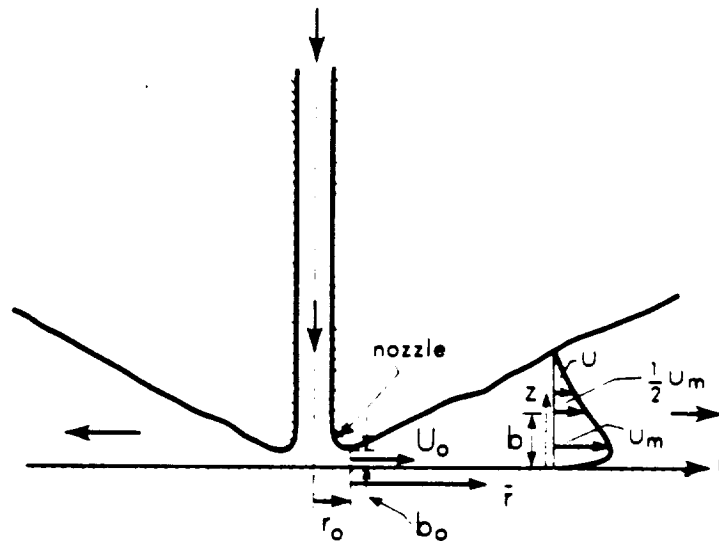


Figure 2.1: Radial wall jet.

An approximate description is found by ignoring the shear forces at the wall in comparison with the momentum of the jet to simplify the analysis.

2.2 Governing Equations

The equations of motion for the turbulent radial jet are as follows (see [6] and [7]):

$$u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} = - \frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right] - \left[\frac{\partial}{\partial r} \overline{u'^2} + \frac{\partial}{\partial z} \overline{u'w'} + \frac{\overline{u'^2}}{r} - \frac{\overline{v'^2}}{r} \right] \quad (2.1)$$

$$u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{uv}{r} = \nu \left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2} \right] - \left[\frac{\partial}{\partial r} \overline{u'v'} + \frac{\partial}{\partial z} \overline{v'w'} + 2 \frac{\overline{u'v'}}{r} \right] \quad (2.2)$$

$$u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right] - \left[\frac{\partial}{\partial r} \overline{u'w'} + \frac{\partial}{\partial z} \overline{w'^2} + \frac{\overline{u'w'}}{r} \right] \quad (2.3)$$

$$\frac{\partial}{\partial r} r u + \frac{\partial}{\partial z} r w = 0 \quad (2.4)$$

where u , v , w are the time mean velocities in the r , ϕ and z directions; u' , v' and w' are the respective velocity fluctuations; p is the pressure; ρ is the density and ν is the kinematic viscosity. With the assumptions that $v = 0$, and $u \gg w$ and transverse gradients, i.e., $\frac{\partial}{\partial z}$ are much larger than longitudinal gradients, i.e., $\frac{\partial}{\partial r}$, the equations of motion for the radial turbulent jet become:

$$u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = \frac{1}{\rho} \frac{\partial \tau}{\partial z} \quad (2.5)$$

$$\frac{\partial}{\partial r}ru + \frac{\partial}{\partial z}rw = 0 \quad (2.6)$$

where $\tau = -\rho \overline{u'w'}$ is the shear stress and the pressure gradient in the radial direction has been assumed to be zero.

It is assumed that

$$\frac{\tau}{\rho} = \varepsilon \frac{\partial u}{\partial z} \quad (2.7)$$

where ε is the eddy viscosity coefficient:

$$\varepsilon = K_1 r u_m \quad (2.8)$$

so that the momentum equation (2.5) becomes

$$u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = K_1 r u_m \frac{\partial^2 u}{\partial z^2} \quad (2.9)$$

2.3 Similarity Velocity Distribution

The integral form of the momentum equation is found, using the continuity equation, as

$$\int_0^{\infty} \rho r u^2 dz = \text{constant} \quad (2.10)$$

where the skin friction has been ignored compared with the shear stress in the jet.

Inspection of (2.9) and (2.10) shows that $u_m \propto \frac{1}{r}$ so that $\varepsilon = \text{constant}$.

Further, dimensionless variables

$$\eta = \frac{1}{\sqrt{2K_1}} \frac{z}{r}; \quad \frac{u}{u_m} = F'(\eta); \quad \frac{w}{u_m} = \sqrt{2K_1} (\eta F' - F) \quad (2.11)$$

are defined so that continuity is satisfied and the momentum equation becomes

$$F''' + 2(F'^2 + FF'') = 0 \quad (2.12)$$

which has the solution

$$F = \tanh \eta \quad (2.13)$$

satisfying the boundary conditions

$$F(0) = 0, \quad F'(0) = 1, \quad F''(0) = 0$$

Defining the half width of the jet as $b = Kr$ (such that $\frac{u}{u_m} = \frac{1}{2}$ when $z = b$), it can be seen that

$$\frac{1}{2} = \text{sech}^2 \left[\frac{1}{\sqrt{2K_1}} K \right] \quad (2.14)$$

$$\text{so that } K_1 = \frac{1}{2} \left[\frac{K}{\text{sech}^{-1} \left(\frac{1}{\sqrt{2}} \right)} \right]^2 = \frac{1}{2} \frac{1}{(.8814)^2} K^2$$

(Experimentally it has been observed (Bakke 1957 [8]) that $K = .078$

$$\text{so that } K_1 = \frac{1}{2} \left[\frac{.078}{.8814} \right]^2 = .004.)$$

It is convenient here to retain $K_1 = .6K^2$. Thus the final solution of u is

$$\frac{u}{u_m} = \text{sech}^2 \left[\frac{.8814}{K} \frac{z}{r} \right] \quad (2.15)$$

2.4 Transport of Dust Particles

The equation for the transport of dust particles by convection and turbulent diffusion is

$$u \frac{\partial c}{\partial r} + w \frac{\partial c}{\partial z} = \varepsilon \frac{\partial^2 c}{\partial z^2} \quad (2.16)$$

with $\varepsilon = K_1 r u_m$, where c is the concentration of dust particles in the flow.

$$\text{Assuming } c = c_w G(\eta) \quad (2.17)$$

where c_w is the concentration at the wall, then equation (2.16) becomes

$$G'' + 2FG' = 0 \quad (2.18)$$

having the solution

$$G = 1 - \tanh \eta \quad (2.19)$$

which satisfies the boundary conditions $G(0) = G(\infty) = 0$.

Thus the concentration may be written

$$c = c_w \left[1 - \tanh \left(\frac{.8814}{K} \frac{z}{r} \right) \right] \quad (2.20)$$

The density ρ of the matrix of air and dust particles is then written

$$\begin{aligned} \rho &= \rho_a (1 - c_w G) + \rho_d c_w G \\ &= \rho_a + (\rho_d - \rho_a) c_w \left[1 - \tanh \left(\frac{.8814}{K} \frac{z}{r} \right) \right] \end{aligned} \quad (2.21)$$

From (2.10), the momentum integral for the wall jet is given by

$$\int_0^{\infty} 2\pi r p u^2 dz = \text{const} = J$$

Evaluation of the integral, using (2.15) and (2.21), gives

$$J = \frac{4\pi}{3} (ru_m)^2 \frac{K}{.8814} \left[\rho_a + (\rho_d - \rho_a) c_w \left(\frac{5}{8} \right) \right] \quad (2.22)$$

The concentration c_w is unknown and is evaluated in terms of the mass transfer at the wall from the boundary condition

$$\left[-\epsilon \frac{\partial c}{\partial z} \right]_w = (1 - c_w) \frac{\dot{m}}{\rho_w} \quad \text{where} \quad \epsilon = K_1 ru_m \quad (2.23)$$

This condition states that convection of air, of concentration $(1 - c_w)$ away from the wall is balanced by diffusion of air toward the wall $\left[-\epsilon \frac{\partial c}{\partial z} \right]_w$.

From (2.23)

$$\frac{\rho_w c_w}{1 - c_w} u_m^2 = \dot{m} u_m \sqrt{\frac{2}{K_1}} = \frac{2 \times .8814}{K} \dot{m} u_m = \frac{1.76}{K} \dot{m} u_m \quad (2.24)$$

However, the momentum acquired by the dust particles is related to the reduction of shear stress at the wall.

If τ_0 is the shear stress of the wall jet in the absence of mass transport and τ^* is the maximum shear stress that the wall can sustain, then

$$\tau_0 - \tau^* = \dot{m} \overline{u} \quad (2.25)$$

where \overline{u} is the average velocity of the dust particles.

The maximum sustainable shear stress is given by the relation

$$\frac{\text{tangential stress}}{\text{normal stress}} = \frac{\tau^*}{\rho_d a_g D c_c} = \tan \alpha \quad (2.26)$$

where c_c is the compaction factor and α is the angle of repose of the dust determined experimentally.

Taking $c_c \cdot \tan \alpha = \frac{1}{2}$, we have

$$\tau^* = \frac{1}{2} \rho_d a_g D$$

The average velocity of the dust particles is found as

$$\bar{u} = \frac{\int_0^\infty u^2 G dz}{\int_0^\infty u G dz} = \frac{5}{6} u_m \quad (2.27)$$

(substituting for $u = u_m F'$ and $c = c_w G$)

and recognizing from (2.22) that

$$\rho_a u_{mo}^2 = \left[\rho_a + (\rho_d - \rho_a) c_w \left(\frac{5}{8} \right) \right] u_m^2 \quad (2.28)$$

$$\text{and } \rho_w = \rho_a + (\rho_d - \rho_a) c_w \quad (2.29)$$

We can write (2.25) in terms of skin friction coefficients

$$C_f = \frac{\tau_o}{\frac{1}{2} \rho_a u_{mo}^2}; \quad C_f^* = \frac{\tau^*}{\frac{1}{2} \rho_a u_{mo}^2} \quad (2.30)$$

and

$$C_f - C_f^* = \frac{\dot{m} \bar{u}}{\frac{1}{2} \rho_a u_{mo}^2}$$

$$= \frac{5}{3} \frac{K}{1.76} \frac{c_w}{1-c_w} \frac{\rho_a + (\rho_d - \rho_a)c_w}{\rho_a + \frac{5}{8}(\rho_d - \rho_a)c_w}$$

$$\approx K \frac{c_w}{1-c_w} \frac{\rho_a + (\rho_d - \rho_a)c_w}{\rho_a + \frac{5}{8}(\rho_d - \rho_a)c_w}$$

$$\text{i.e. } \frac{c_w}{1-c_w} \frac{\rho_a + (\rho_d - \rho_a)c_w}{\rho_a + \frac{5}{8}(\rho_d - \rho_a)c_w} = \mu$$

$$\text{where } \mu = \frac{(C_f - C_f^*)}{K}$$

For $\frac{\rho_d}{\rho_a} \gg 1$ (usually of order $10^3 - 10^4$), this equation can be approximated as

$$\frac{c_w}{1 - c_w} \approx \frac{5}{8}\mu$$

$$\text{or } c_w \approx \frac{\mu}{\mu + \frac{8}{5}}$$

Thus the concentration of the dust particles is given by

$$c \approx \frac{\mu}{\mu + \frac{8}{5}} \left[1 - \tanh\left(\frac{.8814}{K} \frac{z}{r}\right) \right]$$

2.5 Surface Erosion

It can be seen that surface erosion ceases for sufficiently small applied shear stress $\tau_o(r)$, i.e., when $\tau_o < \tau^*$, or when $C_f < \frac{\rho_d a_g D}{\rho_a u_{mo}^2}$.

Thus since $\rho_a u_{m0}^2 = \frac{3}{4\pi} \frac{k}{K} \frac{J}{r^2}$, we find that erosion ceases by

$$r > \left(\frac{J}{\rho_d a_g D} \right)^{\frac{1}{2}} \left(\frac{3k}{4\pi K} C_f \right)^{\frac{1}{2}}$$

or substituting a typical wall stress relation for C_f

$$\text{i.e., } C_f = .0447 \left(\frac{v_a^2}{K^2 r^2 u_{m0}^2} \right)^{\frac{1}{11}}$$

we find that erosion ceases for

$$r > .43 \left(\frac{J}{\rho_d a_g D} \right)^{\frac{1}{2}} \left(\frac{\rho_a v_a^2}{J} \right)^{\frac{1}{22}}$$

(for $k = .8814$ and $K = .078$)

i.e., the radial distance over which erosion takes place varies as

$\left(\frac{J}{\rho_d a_g D} \right)^{\frac{1}{2}}$ but is very weakly dependent on the wall jet Reynolds

number $\left(\frac{J}{\rho_a v_a^2} \right)^{\frac{1}{2}}$ varying as $\left(\frac{J}{\rho_a v_a^2} \right)^{-\frac{1}{22}}$.

Chapter 3

Concluding Remarks

A theoretical model has been established to describe the effects of dust entrainment by a turbulent radial wall jet. The theory suggests scaling laws for the entrainment and gives the concentration of dust particles in the flow. It is shown that entrainment occurs if the applied shear stress exceeds the maximum allowable shear stress that the surface can sustain. Since the applied shear stress for a turbulent radial wall jet decreases with distance from the momentum source, entrainment ceases for sufficiently large distance.

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